IMAGE DENOISING FOR SIGNAL-DEPENDENT NOISE

Keigo Hirakawa[*]

New England Conservatory of Music
Boston, MA 02115
kh237@cornell.edu

Thomas W. Parks

Cornell University
Electrical and Computer Engineering
Ithaca, NY 14850

ABSTRACT

In this paper, we present a method for removing noise from digital images corrupted with additive, multiplicative, and mixed noise. An image patch from an ideal image is modeled as a linear combination of image patches from the noisy image. We propose to fit this image model to the real-world image data in the total least square (TLS) sense, because the TLS formulation allows us to take into account the uncertainties in the measured data. We develop a method to reduce the contribution from the irrelevant image patches, which will sharpen the edges and reduce edge artifacts at the same time. Although the proposed algorithm is computationally demanding, the image quality of the output image demonstrates the effectiveness of the TLS algorithms.

1. INTRODUCTION

In real-world digital imaging devices, the images we are interested in often are corrupted by device-specific noise. Basic research in image denoising, therefore, would prove useful to applications such as low-light imaging and lossy compression. CMOS and CCD sensors are two very important special cases of imaging devices that suffer from noise. In CMOS sensors, we see a fixed-pattern noise, and a mixture of independent additive and multiplicative Gaussian noise [13]:

$$x = s + (k_0 + k_1 s)\delta, \tag{1}$$

where k_0 and k_1 are constants, and $\delta \sim \mathcal{N}(0,1)$. We independently confirmed that (1) is a good noise model for Agilent Technology's consumer CMOS digital camera. While effective methods to remove fixed-pattern noise have been proposed [10], removing noise of the form (1) proves difficult. Many papers in the literature, however, prefer a simpler noise model [6] [7] [8] [9] [11] [12]:

$$x = s + k_0\delta. \tag{2}$$

Note that (2) is a special case of (1).

In the recent literature, statistical modeling of wavelet coefficients has been popular [1] [4] [6] [8] [9] [11] [12]. The study of inter-dependencies of wavelet coefficients across scale, especially, has gained strong momentum, and pair-wise processing of a coefficient and its parent is common. While wavelets share some behavioral characteristics with the neurological response of a human eye, in most cases the statistical modeling of wavelets have been derived heuristically.

We develop a model relating the noisy image to an ideal image in the total least squares (TLS) sense, taking into account the stochastic nature of the noise and allowing small perturbations in the system. Furthermore, we develop a denoising algorithm that, while effective in removing noise of the form (2), removes the signal-dependent noise of the form (1).

This paper is organized as follows. In section 2, we present our deterministic image model, and introduce the basics of TLS denoising algorithm. Generalizations of the algorithm are made in section 3. In section 4 we compare results with the state-of-the-art denoising algorithms.

2. TLS IMAGE DENOISING THEORY

In this section, we introduce the TLS image denoising theory at its basic level. In the image denoising problem, only the noisy image data is observed. We develop an image model relating the noisy image to a clean image based on the TLS framework (section 2.1). We solve this TLS problem for the case that an image is corrupted by signal-dependent noise (section 2.2).

2.1. Simple TLS Image Model

Suppose we are given an ideal clean image, s, and a noise corrupted version, x. Let $s_0 \in \mathbb{R}^m$ be an image patch from s (i.e. $\sqrt{m}\times\sqrt{m}$ vector cropped from an image) and $\{x_i \in \mathbb{R}^m\}_{i\in\{1,\ldots,n\}}$, $m \geq n+1$ be a collection of image patches from x that are *reasonably* similar to s_0. To find the relationship between s_0 and the noisy image, x, we would like to represent s_0 as a linear combination of $\{x_i\}$:

$$s_0 = X\alpha, \tag{3}$$

where $X = [x_1, \ldots, x_n]$. However, in general there is no such α that makes (3) true because $s_0 \notin \texttt{span}\{x_i\}$. Suppose we allow a small perturbation e_0 in the system so that

$$s_0 + e_0 = X\alpha. \tag{4}$$

The vector α that that satisfies (4) with the smallest perturbation e_0 in the L^2 sense is commonly known as the least square (LS) solution. However, the inherent flaw in the above system is that the perturbation is confined to s_0, even though there is noise in X.

Instead, we propose to allow small perturbations in both s_0 and X:

$$s_0 + e_0 = (X + E)\alpha. \tag{5}$$

The vector α satisfying (5) while minimizing $\|[E, e_0]\|_F^2$ is known as the total least square solution, denoted $\alpha_{\texttt{TLS}}$. Here, $\|\cdot\|_F$ is

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the Frobenius norm. In general, the perturbation in X makes the perturbation in s_0 smaller.

The solution to (5) is well documented [2] [5]. First, examine $[X, s_0]$ using singular value decomposition (SVD) $[X, s_0] = U\Sigma V^T$, where $\Sigma = \texttt{diag}(\sigma_1, \ldots, \sigma_{n+1})$, $\sigma_i^2 > \sigma_{i+1}^2$. Then

$$\alpha_{\text{TLS}} = -[v_{1,n+1}, \ldots, v_{n,n+1}]^T v_{n+1,n+1}^{-1} \tag{6}$$

where $[v_{1,n+1}, \ldots, v_{n+1,n+1}]^T$ is the left and right singular vectors corresponding to σ_{n+1}, respectively.

2.2. TLS Solution with Signal-Dependent Noise

The solution to (6) requires the knowledge of the clean image patch s_0, but this is not available in a denoising problem. In this section, we develop a method to compute α_{TLS}, where an image corrupted by signal-dependent noise is given, and s_0 is not provided. More specifically, assume (1). Define s_i as an image patch from s corresponding to x_i, and assume $s_0 \in \{s_i\}$. Then

$$x_i = s_i + k_0\delta_i + k_1\texttt{diag}(s_i)\delta_i, \tag{7}$$

where $\delta_i \in \mathbb{R}^m$ is a noise vector, and $diag(s_i)$ is a diagonal matrix whose diagonal entries are the entries of s_i.

We solve for α_{TLS} without s_0 and taking into the account the stochastic nature of δ_i. Consider the following:

$$P = [X, s_0]^T[X, s_0] = (U\Sigma V^T)^T(U\Sigma V^T) = V\Sigma^2 V^T,$$

where $[X, s_0] = U\Sigma V^T$ is SVD. Our strategy is to estimate P and obtain the right singular vector V through its eigen decomposition.

Define $\mathcal{E}\{\cdot\}$ as the expectation operator. Estimating P is rather simple. When $m \gg n+1$, $P \approx \mathcal{E}\{P\}$, so

$$\begin{aligned} P &= \mathcal{E}\{[X, s_0]^T[X, s_0]\} \\ &= \begin{bmatrix} \mathcal{E}\{X^TX\} & \mathcal{E}\{X^Ts_0\} \\ \mathcal{E}\{s_0^TX\} & \mathcal{E}\{s_0^Ts_0\} \end{bmatrix} = \begin{bmatrix} P_{XX} & S^Ts_0 \\ s_0^TS & s_0^Ts_0 \end{bmatrix}, \end{aligned} \tag{8}$$

where $P_{XX} = \mathcal{E}\{X^TX\}$. When we assume

$$\mathcal{E}\{\delta_i\} = 0, \quad \mathcal{E}\{\delta_i\delta_j\} = \begin{cases} I & i = j \\ 0 & i \neq j \end{cases}, \tag{9}$$

P_{XX} simplifies to:

$$\begin{aligned} P_{XX} &= S^TS + mk_0^2I + k_1^2\sum_{i=1}^{m}\texttt{diag}(s_{i,1}^2, \ldots, s_{i,n}^2) \\ &\quad + 2k_0k_1\sum_{i=1}^{m}\texttt{diag}(s_{i,1}, \ldots, s_{i,n}). \end{aligned}$$

When $m \gg n+1$, we can also approximate $\sum_i s_{i,j}$ as $\sum_i x_{i,j}$, which is computable. Therefore, using the fact that the jth diagonal entry of S^TS is $\sum_{i=1}^m s_{i,j}^2$, S^TS can be estimated using the following procedure:

1. Compute $P_{XX} = X^TX$.
2. Compute $P_{XX} - k_0^2mI - 2k_0k_1\sum_i \texttt{diag}(x_{i,1}, \ldots, x_{i,n})$.
3. Multiply diagonal entries of $(P_{XX} - k_0^2mI)$ by $(1+k_1^2)^{-1}$.

S^Ts_0, s_0^TS and $s_0^Ts_0$ can be estimated by taking the appropriate rows and columns from the above S^TS estimate. Therefore, the matrix P is fully computable. The new α_{TLS} is computed from (6), where $[v_{1,n+1}, \ldots, v_{n+1,n+1}]^T$ is the eigen vector corresponding to the smallest eigen value of P. Our best estimate for s_0 is $\hat{s}_0 = X\alpha_{\text{TLS}}$.

3. ENHANCEMENTS TO TLS IMAGE MODEL

In this section, we offer a number of different generalizations to the TLS image models developed in section 2. Given the page constraints, the sections 3.1-3.4 give high-level descriptions only. Their mathematical details will be presented in section 3.5 in a combined form. In some cases, variables are redefined to match the improved behaviors of these generalized algorithms.

3.1. Affine Approximation

A variation to the TLS problem (5) using an affine approximation model was solved by de Groen [2]. He showed that $\|[E, e]\|_F^2 = \sigma_{n+1}^2$ is reduced greatly when the column-means of $[X, s_0]$ are subtracted from their respective columns first, suggesting a better model fit. More specifically, instead of (5), we solve for α in the following system that minimizes $\|E, e_0\|_F^2$:

$$\tilde{s}_0 + e_0 = (\tilde{X} + E)\alpha$$

where $\tilde{s}_0 = s_0 - \bar{s}_0$, $\tilde{x}_i = x_i - \bar{x}_i$ (ith column of $\tilde{X}$), and $\bar{s}_0, \bar{x}_i \in \mathbb{R}$ are the average values of elements in s_0 and x_i, respectively.

3.2. Image Patch Selection

In section 2.1, we described $\{x_i\}$ as a collection of image patches that are *reasonably* similar to s_0. In order for our image model (5) to be effective, the set $\{x_i\}$ must be chosen such that image features in s_0 are well captured. The first approach is to take the $\sqrt{m} \times \sqrt{m}$ vectors cropped from the noisy image x in the spatial vicinity of s_0 [7] (call this set $\{x_i^{(1)}\}$). The second approach, which is motivated by multi-resolution analysis and self-similarity properties in a natural image, is to take the $\sqrt{m} \times \sqrt{m}$ vectors from a decimated image, in the spatial vicinity of s_0 (call this set $\{x_i^{(2)}\}$)

3.3. Adaptive Weights

There will inevitably be some image patches in $\{x_i^{(1)}\}$ and $\{x_i^{(2)}\}$ that resemble s_0 in limited regions only. The use of the weighting matrices can help aid the TLS denoising algorithm by giving more weight to the pixels that collectively describe the image structure in the center region of s_0. Let $A = \texttt{diag}(a_1, \ldots, a_m)$, $B = \texttt{diag}(b_1, \ldots, b_{n+1})$, A and B non-singular. The TLS image model can be modified so that α is chosen to satisfy (5) while minimizing $\|A[E, e_0]B\|_F^2$ instead of $\|[E, e_0]\|_F^2$. Notice that A (B) scales the rows (columns) of $[E, e_0]$.

Owing to the techniques developed for bilateral filtering [14], *range* distance metrics is used to determine A and B adaptively:

$$\begin{aligned} a_i &= \exp\left(-\texttt{dist}_A([x_{i,1}, \ldots, x_{i,n}], [x_{c,1}, \ldots, x_{c,n}])^2/k_A\right) \\ b_j &= \begin{cases} \exp\left(-\texttt{dist}_B(x_j, x_0)^2/k_B\right), & \forall j \leq n \\ \gamma & \forall j > n \end{cases} \end{aligned}$$

where γ, k_A, k_B are constants, $\texttt{dist}_A$ and $\texttt{dist}_B$ are range distance functions, and $[x_{c,1}, \ldots, x_{c,n}]$ is the row in X corresponding to the center pixel of $\sqrt{m} \times \sqrt{m}$ image patch. Intuitively, a_i and b_j measure the similarity between the pair of given vectors. In the results presented in this paper, we use:

$$\begin{aligned} \texttt{dist}_A(\phi, \psi) &= \|\phi - \psi\|_2 \\ \texttt{dist}_B(\phi, \psi) &= \|H(\phi - \psi)\|_2, \end{aligned}$$

Table 1. Denoising methods evaluated using SSIM. Images corrupted by noise generated by $(k_0, k_1) = (25,0), (25,0.1), (25,0.2)$, respectively.

	noisy	proposed	[9]	[8]	[11]	[7]
Lena	0.2734	0.8528	0.8514	0.8446	0.8397	0.8278
	0.1784	0.8228	0.8169	0.8066	0.7989	0.7803
	0.1301	0.7969	0.7790	0.7688	0.7483	0.7269
Barbara	0.4055	0.8657	0.8420	0.8213	0.8379	0.8435
	0.2888	0.8079	0.7737	0.7396	0.7739	0.7765
	0.2181	0.7555	0.7073	0.6664	0.7081	0.7084
Boats	0.3494	0.7816	0.7856	0.7790	0.7724	0.7567
	0.2293	0.7199	0.7275	0.7187	0.7085	0.6858
	0.1677	0.6742	0.6752	0.6668	0.6479	0.6218
House	0.2799	0.8378	0.8319	0.8293	0.8045	0.8131
	0.1818	0.8086	0.7981	0.7898	0.7649	0.7709
	0.1300	0.7824	0.7609	0.7491	0.7156	0.7183
Peppers	0.3542	0.8451	0.8427	0.8510	0.8171	0.8170
	0.2472	0.8050	0.7955	0.8021	0.7608	0.7590
	0.1887	0.7737	0.7514	0.7587	0.7053	0.7035
F Prints	0.6939	0.9030	0.9038	0.8922	0.9066	0.8897
	0.5168	0.8469	0.8499	0.8302	0.8505	0.8278
	0.3920	0.7779	0.7927	0.7588	0.7879	0.7498

where $H = \texttt{diag}(h_1, \ldots, h_m)$ and $[h_1, \ldots, h_m]$ is a Gaussian envelope centered at the center of the $\sqrt{m} \times \sqrt{m}$ image patch. H is needed because $m \gg n$ is large.

3.4. Redundant Estimation

Let $S_0 = [s_1, \ldots, s_p]$, where $\{s_i\}$ is a collection of image patches from s. Then our new TLS system is modified as follows:

$$S_0 + E_0 = (X + E)\alpha, \tag{10}$$

where the perturbation E_0 is now $m \times p$, and $\alpha \in \mathbb{R}^{n \times p}$. A matrix α satisfying (10) while minimizing $\|A[E, E_0]B\|_F^2$ is known as the TLS solution, denoted $\alpha_{\texttt{TLS}}$. The solution to (10) is well documented [2] [5].

Working with (10) has several advantages over (5). First, by choosing to minimize the perturbation in multiple image patches $\{s_i\}$ simultaneously, the algorithm becomes more robust against noise. To see this, note that $A[E, E_0]B$ is rank p [5], which offers more freedom over the perturbation than $A[E, e_0]B$ allows. This is in a sharp contrast to the analogous LS system, $S_0 + E_0 = X\alpha$, because the LS solution that minimizes $\|E_0\|_F^2$ will be no different than if each columns of E_0 were minimized independently. Second, assuming that $\{s_1, \ldots, s_p\}$ were picked from the same region of the image s, there will be overlapping regions in the denoised image patches. We benefit from this by combining some or all of estimated pixel values that are available at each position. With this technique, the edge artifacts are reduced and smooth surfaces become significantly smoother, while the sharpness of the edges is preserved.

3.5. Mathematical Details

In this section, we develop a method to compute $\alpha_{\texttt{TLS}}$ from an image corrputed by signal-dependent noise that incorporates techniques in sections 3.1-3.4. We begin with (7) and (9). Define $\bar{x}_j = (\sum_i a_i^2 x_{i,j})/(\sum_i a_i^2)$, $\tilde{x}_{i,j} = x_{i,j} - \bar{x}_j$, $\tilde{X} = [\tilde{x}_1, \ldots, \tilde{x}_n]$; define $\bar{s}_j$,$\tilde{s}_j$,$\tilde{S}$ similarly; let $\tilde{S}_0 = [\tilde{s}_1, \ldots, \tilde{s}_p]$. Let $A[\tilde{X}, \tilde{S}_0]B = U\Sigma V^T$ be SVD, where $\Sigma = \texttt{diag}(\sigma_1, \ldots \sigma_{n+p})$ and $\sigma_i^2 > \sigma_{i+1}^2$. Partition U and V as

$$U = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \quad V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \begin{matrix} n \\ p \end{matrix} .$$
$$\qquad\; n \quad p \qquad\qquad\qquad n \quad p$$

Then the value for α that minimizes $\|A[E, E_0]B\|$ while satisfying $\tilde{S}_0 + E_0 = (\tilde{X} + E)\alpha$ is

$$\alpha_{\texttt{TLS}} = -B_1 V_{12} V_{22}^{-1} B_2^{-1}. \tag{11}$$

where $B_1 = \texttt{diag}(b_1, \ldots, b_n)$, $B_2 = \texttt{diag}(b_{n+1}, \ldots, b_{n+p})$.

Given the noisy image x, the general TLS problem can still be solved without $\tilde{S}_0$. To see this, consider

$$P = (A[\tilde{X}, \tilde{S}_0]B)^T (A[\tilde{X}, \tilde{S}_0]B) = V\Sigma^2 V^T. \tag{12}$$

For $m \gg n + p$, $P \approx \mathcal{E}\{P\}$, and

$$\begin{aligned} P &= \mathcal{E}\{(A[\tilde{X}, \tilde{S}_0]B)^T (A[\tilde{X}, \tilde{S}_0]B)\} \\ &= B \begin{bmatrix} P_{XX} & S^T A^2 S_0 \\ S_0^T A^2 S & S_0^T A^2 S_0 \end{bmatrix} B \end{aligned}$$

where $P_{XX} = \mathcal{E}\{\tilde{X}^T A^2 \tilde{X}\}$. Let us assume (9), and that for $m \gg n + p$, $\bar{x}_j \approx \bar{s}_j$. Then P_{XX} simplifies to

$$\begin{aligned} P_{XX} = \tilde{S}^T A^2 \tilde{S} + k_1^2 \sum_{i=1}^{m} a_i^2 \texttt{diag}(\bar{s}_{i,1}^2, \ldots, \bar{s}_{i,n}^2) \\ + \texttt{diag}((k_0 + k_1\bar{x}_1)^2, \ldots, (k_0 + k_1\bar{x}_n)^2) \left(\sum_{i=1}^{m} a_i^2\right) \end{aligned}$$

Since the jth diagonal entry of $\tilde{S}^T A^2 \tilde{S}$ is $\sum_i a_i^2 \tilde{s}_{i,j}^2$, $\tilde{S}^T A^2 \tilde{S}$ can be estimated using the following procedure:

1. Compute $P_{XX} = \tilde{X}^T A^2 \tilde{X}$.
2. Compute $P_{XX} - \texttt{diag}((k_0 + k_1\bar{x}_1)^2, \ldots, (k_0 + k_1\bar{x}_n)^2)(\sum_i a_i^2)$.
3. Multiply diagonal entries of matrix in step 2 by $(1 + k_1^2)^{-1}$.

The first p rows of $\tilde{S}^T A^2 \tilde{S}$ is $\tilde{S}_0^T A^2 \tilde{S}$, and the top-left $p \times p$ submatrix of $\tilde{S}^T A^2 \tilde{S}$ is $\tilde{S}_0^T A^2 \tilde{S}_0$. Thus the matrix P is fully computable. The new $\alpha_{\texttt{TLS}}$ is computed from (11) where V is given by the eigen decomposition of P in (12). Our best estimate for S_0 is

$$\hat{S}_0 = \tilde{X}\alpha_{\texttt{TLS}} + [1, \ldots, 1]^T [\bar{x}_0, \ldots, \bar{x}_p].$$

3.6. Pre-Processing

The effectiveness of the TLS denoising algorithm depends on our ability to estimate P matrix accurately. Given $\delta \sim \mathcal{N}(0,1)$, there will be one or two pixels occasionally that stand out because the value of δ at that pixel position is far greater than its standard deviation. This is problematic because the entries in X appear more than once, degrading our estimate for P greatly.

To work around this problem, we propose to prune the outliers. The following pre-processing procedure was used. For each pixel location in x,

1. Crop a 5×5 vector from x. We will call it y.
2. Find the Nth largest and Nth smallest pixel values in y.
3. If the center pixel in y is larger (smaller) than the Nth largest (smallest) pixel value in y, replace the center pixel value with the Nth largest (smallest) pixel value in y.

Fig. 1. Example cropped from "Lena" with noise $(k_0, k_1) = (25, 0.1)$. Output from method in [9] (left) and proposed algorithm (right).

4. IMPLEMENTATION AND RESULTS

Our TLS algorithm is implemented with $m = 23 \times 23 = 529$, $n_1 = 5 \times 5 = 25, n_2 = 5 \times 5 = 25$, where n_1, n_2 are the numbers of vectors in $\{x^{(1)}\}$ and $\{x^{(2)}\}$, respectively. The columns of $\tilde{S}_0$ are the image patches in s corresponding to $\{x_i^{(1)}\}$. Eigen decomposition of P, which requires $O((2n_1 + n_2)^2)$ operations, is the most computationally intensive procedure in the algorithm. We compared our method to works published recently [7] [8] [9] [11]. Experiments are performed on well-known 8-bit gray-scale test images. Parameters k_0 and k_1 were available *a priori* to all algorithms. In table 1, performance is evaluated using structural similarity index (SSIM) [15], which is a better measure of image quality than PSNR. Because [7] [9] [8] [11] assume the noise model in (2), generalized homomorphic filtering is used to approximately decouple the noise from the signal [3] before denoising; an inverse filter is applied after denoising. SSIM values show that the proposed method is comparable to the state-of-the-art denoising methods when $k_1 = 0$, and is an improvement when $k_1 \neq 0$. Fig. 1 shows an example output when $(k_0, k_1) = (25, 0.1)$. The proposed algorithm preserves the details of the feathers on the hat, and smoothes the homogeneous regions (e.g. cheeks and background).

5. CONCLUSION

In this paper, a new image denoising algorithm based on TLS techniques was presented. An ideal image patch was modeled as a linear combination of vectors cropped from the noisy image, and we fit the model to the real image data by allowing a small perturbation in the TLS sense. A new technique to solve the TLS problem without the knowledge of the ideal image patch when the image is corrupted by signal-dependent noise is developed. The output images from the proposed algorithm showed improved image quality, when compared to recently published work. Future research in this field includes reduction of computational complexity and a more sophisticated weighting scheme.

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